Find the point-slope form of the equation of the **normal** line to the curve $y = x^3 \sec x$ at the point where $x = \pi$. SCORE: _____ / 4 PTS

$$\frac{dy}{dx} = \boxed{3x^2 \sec x + x^3 \sec x \tan x} \; 2$$

WHEN $x = \pi$,

$$\left.\frac{dy}{dx}\right|_{x=\pi} = 3\pi^2(-1) + \pi^3(-1)(0) = \boxed{-3\pi^2} \; 1$$

$$y = \pi^3(-1) = -\pi^3$$

$$\boxed{y + \pi^3 = \frac{1}{3\pi^2}(x - \pi)} \; 1$$

If $f(2) = 4$ and $f'(2) = -1$ and $g(x) = x^3 f(x)$, find $g'(2)$. SCORE: _____ / 3 PTS

$$g'(x) = \boxed{3x^2 f(x) + x^3 f'(x)} \; 2$$

$$g'(2) = 3(2)^2 f(2) + 2^3 f'(2)$$

$$= 12(4) + 8(-1)$$

$$= \boxed{40} \; 1$$

Prove the derivative of $\sec x$ using the quotient rule. **Show all steps.** SCORE: _____ / 3 PTS

$$\frac{d}{dx} \frac{1}{\cos x} = \boxed{\frac{0 \cdot \cos x - 1 \cdot (-\sin x)}{\cos^2 x}} = \boxed{\frac{\sin x}{\cos^2 x}} = \boxed{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}} \; \frac{1}{2}$$

1 1

$$= \boxed{\sec x \tan x} \; \frac{1}{2}$$

The position of an object at time t is given by $s(t) = 2\cos t - 4\sin t$.

Find the acceleration of the object at time $t = \frac{\pi}{3}$. SCORE: _____ / 3 PTS

$$s'(t) = \boxed{-2\sin t - 4\cos t} \; 1$$

$$s''(t) = \boxed{-2\cos t + 4\sin t} \; 1$$

$$s''\left(\frac{\pi}{3}\right) = -2\left(\frac{1}{2}\right) + 4\left(\frac{\sqrt{3}}{2}\right) = \boxed{-1 + 2\sqrt{3}} \; 1$$

Prove the quotient rule for derivatives using the definition of the derivative function. **SCORE: _____ / 5 PTS**

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}$$

$$= \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}$$

$$= \lim_{h\to 0}\frac{\frac{f(x+h)-f(x)}{h}g(x) - f(x)\frac{g(x+h)-g(x)}{h}}{g(x+h)g(x)}$$

$$= \frac{\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}\lim\limits_{h\to 0} g(x) - \lim\limits_{h\to 0} f(x)\lim\limits_{h\to 0}\frac{g(x+h)-g(x)}{h}}{\lim\limits_{h\to 0} g(x+h)\lim\limits_{h\to 0} g(x)}$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Find the following derivatives. Simplify all answers appropriately. **SCORE: _____ / 12 PTS**

[a] $\frac{d^2}{dx^2}\frac{(2x-5)^2}{\sqrt{x}} = \frac{d^2}{dx^2}\frac{4x^2-20x+25}{x^{\frac{1}{2}}}$

$= \frac{d^2}{dx^2}(4x^{\frac{3}{2}}-20x^{\frac{1}{2}}+25x^{-\frac{1}{2}})$

$= \frac{d}{dx}(6x^{\frac{1}{2}}-10x^{-\frac{1}{2}}-\frac{25}{2}x^{-\frac{3}{2}})$ 2

$= 3x^{-\frac{1}{2}}+5x^{-\frac{3}{2}}+\frac{75}{4}x^{-\frac{5}{2}}$ 1

[b] $\frac{d}{db}\frac{2b-b^2}{3b+1}$ (Your final answer must be a single fraction)

$= \frac{(2-2b)(3b+1)-(2b-b^2)(3)}{(3b+1)^2}$ 2 1

$= \frac{6b+2-6b^2-2b-6b+3b^2}{(3b+1)^2}$

$= \boxed{\frac{-3b^2-2b+2}{(3b+1)^2}}$ 1

[c] $\frac{d}{dy}(3e^x+\frac{1}{y^e}+5e^y)$

$= \frac{d}{dy}(3e^x+y^{-e}+5e^y)$

$= \underbrace{-ey^{-e-1}}_{1}\underbrace{+5e^y}_{1}$

+1 IF NO OTHER TERMS (IE. NO $3e^x$)

[d] $\frac{d}{dx}\frac{\cot x-\sin x}{\cos x} = \frac{d}{dx}(\csc x-\tan x)$

$= \underbrace{-\csc x\cot x}_{1}\underbrace{-\sec^2 x}_{1}$